

Sub-barrier fusion of intermediate and heavy nuclear systems

V.M. Shilov

Bogoliubov Lab. Theor. Phys., 141980, JINR, Dubna, Russia

Abstract

A potential model to describe the total cross section of nuclear fusion reactions at low energies is proposed. It is shown that within an approach with a simple, single barrier potential a satisfactory description of data is hindered, while with models with more complex interactions, e.g., with two-barrier potentials, a good description of data can be achieved in a large interval of colliding nuclei and center of mass energies. In particular, the two-barrier model allows to describe data at low, sub-barrier energies where data exhibits a steep falloff of the cross section. It is also shown that the position of the two barriers is almost independent up on masses of colliding nuclei. Comparison with available experimental data is presented as well.

I. INTRODUCTION

Investigation of fusion reactions at low center of mass energies ($E \simeq 30 - 100 \text{ MeV}$) is of interests not only in context of study of the mechanism of nucleus-nucleus interaction, but also supplies unique information on astrophysical processes on stars, in particular on the so-called astrophysical S -factor, which is directly related with the e.g., flux of solar neutrinos. Experimental measurements at energies well above the Coulomb barrier started a few decades ago [1, 2] and shown that the cross section exhibits a smooth behaviour as a function of the center of mass energy. However, measurements of very small cross sections ($\sigma_{ER} \approx 10^{-2} \text{ mb}$), near and below the Coulomb barrier [3–9] have shown an unexpected steep falloff of the cross section, named as "hindrance" effect. Theoretical efforts to describe such a shape generated a number of models [9–15] within which different reaction mechanisms have been suggested to reconcile data at extreme sub-barrier energies. Since different models describe data equally well, in order to distinguish the proposed mechanisms one needs more measurements at even lower energies; a comparison with model predictions will allow to enlighten the peculiarities of the reaction mechanism.

In the present paper a further development of the so-called "critical distance model" [1, 2] is presented. Remained, that within such a model, in order to create the compound nucleus, the colliding nuclei have to approach some "critical distance" R_{cr} with $R_{cr} = r_{cr}^0 (A_1^{1/3} + A_2^{1/3})$ ($r_{cr}^0 \approx 1 \text{ fm}$). At such distances the colliding nuclei lose their individual structure forming a single highly excited system. Since at the initial stage the collision is mainly superficial the deformation nuclei can be neglected and, correspondingly, the ion-ion potential can be calculated within the so-called "frozen approximation" [1, 2]. It should be noted that when the density in the overlap region becomes larger than equilibrium density, the Pauli principle forbids further penetration of nuclei in to each other. This gives rise of a repulsion part in the ion-ion potential [16, 17] at such critical distances. At smaller distances $r < R_{cr}$ the colliding nuclei forming the compound system become deformed so that the densities cannot be considered "frozen" anymore. This leads to a drastic attenuation of the potential up to values $Q_R = (M_{CN} - M_1 - M_2) c^2$, where Q_R is the threshold energy of the reaction and M denotes the corresponding mass of the nuclear system. It is clear, that if the critical distance R_{cr} is smaller than typical positions of the Coulomb barrier, this results in a two barrier potential; besides the Coulomb barrier here appears another, inner barrier $V_C(R_{cr})$. The Coulomb

barrier dominates at small values of atomic numbers Z , however with increasing of Z the first, inner barrier becomes similar and even larger than the Coulomb barrier. Usually the strength of potentials $V_C(R_{cr})$ is characterized by a parameter η_{cr} ($\eta = Z_1 Z_2 / (A_1^{1/3} A_2^{1/3})$) [18] which demonstrates that already at $\eta_{cr} > 71.3$ the first barrier, $V_C(R_{cr})$, becomes larger than the Coulomb one, $V_B(R_B)$. For light and intermediate nuclei $V_C(R_{cr})$ is always smaller than $V_B(R_B)$ and it contributes only at large angular momenta and high energies.

The aim of this work is to investigate the effects of the two barriers in the ion-ion potential and to explain the observed hindrance as effects of resonance tunneling through such a two-humped potential [10]. It will be shown that the inner barrier manifests also in deep sub-barrier cross sections of medium magic and semi-magic nuclei even at $\eta < \eta_{cr}$. Particular attention is paid to fusion reactions of nuclei with mass number closed to the magic ones. In this case according to the nuclear shell model the single particle energy levels are closed and the nucleus acts as a hard sphere impeding the deformation during the fusion process. As a result, the tunneling of undeformed ions occurs without forming a compound system and the interference between two barriers can become important. This effect is studied in the present paper in details.

Since the procedure of finding the exact solution of the Schrödinger equation (actually, this is a system of coupled equations) is a cumbersome and rather tedious task, in the present approach a semiclassical approximation is adopted within which the system is considered uncoupled and each equation solved separately. The resulting solution contains a number of free parameters which determine the general form of the inner potential. These parameters are to be found from a combined fit of the experimental data for different pairs of colliding nuclei. The possibility to apply the proposed model to other kind of processes with heavy ions is discussed as well.

II. FORMALISM

The equations for radial wave functions $R_{\alpha L_\alpha}(r)$, describing the relative motion of the two ions in the coupled channels method can be written as [19]

$$\left[\frac{d^2}{dr^2} + \frac{2\mu}{\hbar^2} (E - \varepsilon_\alpha - V_\alpha(r)) - \frac{L_\alpha(L_\alpha + 1)}{r^2} \right] R_{\alpha L_\alpha}(r) = \frac{2\mu}{\hbar^2} \sum_\beta V_{\alpha\beta}(r) R_{\beta L_\beta}(r). \quad (1)$$

Here L_α is the orbital angular momentum, $V_\alpha(r)$ is the diagonal part of ion-ion potential in the α channel with excitation energy ε_α and $V_{\alpha\beta}(r)$ is the coupling potential between channels. Eventually, μ is the reduced mass of the system.

As already mentioned, the coupled system (1) can be decoupled and solved by considering the semiclassical approximation [20]. Within this approximation the system is spinless for both channels and the energy levels are considered as a two-level degenerated system in the ground state. Introducing the notation $\chi_L^{(\pm)}(r) = R_{0L}(r) \pm R_{1L}(r)$ and using the equalities $V_0(r) = V_1(r)$ and $V_{01}(r) = V_{10}(r)$, one obtains

$$\left[\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + E - \frac{L(L+1)\hbar^2}{2\mu r^2} - (V_0(r) \pm V_{01}(r)) \right] \chi_L^{(\pm)}(r) = 0. \quad (2)$$

In the simplest case when the fusion process is supposed to be defined as a conserved flux of ions tunneling throughout the Coulomb barrier the system can be solved analytically resulting in the known Wong formulae [21]. Then in this case the fusion cross section can be expressed as

$$\sigma_{fus}(E) = \frac{\hbar\omega R_B^2}{4E} \left\{ \ln[1 + \exp \frac{2\pi}{\hbar\omega} (E - V_0(R_B) - V_{01}(R_B))] \right\} + \frac{\hbar\omega R_B^2}{4E} \left\{ \ln[1 + \exp \frac{2\pi}{\hbar\omega} (E - V_0(R_B) + V_{01}(R_B))] \right\}. \quad (3)$$

In (3) the radii in both channels have been taken the same.

In solving the equation (2) the quasi-classical method WentzelKramersBrillouin (WKB), originally proposed to describe fission reaction [22, 23], has been used. Accordingly, the penetrability T can be written

$$T = T_C T_B / (\{1 + [(1 - T_C)(1 - T_B)]^{1/2}\}^2 \cos^2 \nu + \{1 - [(1 - T_C)(1 - T_B)]^{1/2}\}^2 \sin^2 \nu), \quad (4)$$

where for the sake of brevity in the above equation the angular momentum L dependence has been suppressed; the subscripts B and C stand for the Coulomb and inner barriers, respectively. The corresponding penetrability $T_{C,B}$ within the WKB-approximation at sub-barrier energies is

$$T_{C,B}(E) = \{1 + \exp [2 S_{C,B}(E)]\}^{-1}, \quad (5)$$

where the actions $S_{C,B}(E)$ and phases ν are defined by integrals over the interval between corresponding return points in ion-ion potentials $V_L(r)$

$$S_{C,B}(E) = \int \left| 2\mu [V_L(r) - E] / \hbar^2 \right|^{1/2} dr \quad (6)$$

$$\nu = \int \left| 2\mu [E - V_L(r)]/\hbar^2 \right|^{1/2} dr. \quad (7)$$

At larger penetrability, $T_{C,B} > 0.45$, the tunneling probabilities can be parametrized analytically by known Hill-Wheeler formulae with near-barrier parameters of curvatures

$$\hbar\omega_{C,B} = \left| \frac{\hbar^2}{\mu} \frac{d^2 V_{C,B}(r)}{dr^2} \right|_{r=R_{C,B}}^{1/2}. \quad (8)$$

$$T_{C,B}(E) = \left\{ 1 + \exp \left[\frac{2\pi}{\hbar\omega_{C,B}} (E - V_{C,B}) \right] \right\}^{-1}. \quad (9)$$

Here $V_{C,B}$, $R_{C,B}$ are potentials and radii of inner and Coulomb barriers of each partial wave.

Then the averaged, over ν in eq. (4), penetrability can be written as

$$T = T_C T_B / [1 - (1 - T_C)(1 - T_B)]. \quad (10)$$

Exactly the same expression for the penetrability can be obtained by summing ingoing and outgoing fluxes in the interval between the two barriers. Note, that by averaging in obtaining (10) all the interference effects have been lost. So that a comparison of results by unaveraged formula (4) and by equation (10) will reflect the role of interference effects in reactions of heavy ion fusion at low energies.

III. DATA ANALYSIS

We have analyzed fusion reaction for a variety of combinations of colliding ions available for experimental studies. The corresponding reactions together with some their specific characteristics (Q -reaction, minimal energy, Bass potential and angular momentum of the evaporation residue) are listed in Table I. In our approach these quantities serve as input parameters. Our analysis consists on changing the form of the of ion-ion potential and computing the fusion cross section with further fit of the potential parameters to obtain a best description of data. As the outer part of the Coulomb potential the Wood-Saxon form with parameters V, R, a has been used. The coupled channel potential has been taken in the form

$$V_{01}(r) = \frac{\beta_\alpha R_0}{\sqrt{4\pi}} \left[-\frac{dV_0(r)}{dr} + \frac{3}{2\lambda + 1} Z_1 Z_2 e^2 R_0^{\lambda-1} / r^{\lambda+1} \right], \quad (11)$$

TABLE I: The considered reactions and the relevant parameters used in the numerical calculations. Q_R is the threshold energy of the reaction, E_{min} is the minimal energy at which the experimental measurements have been performed, J_{ER} is the parameter used to distinguish the two deexcitation process, eq. (15), V_{Bass} is the Bass potential used in most phenomenological calculations.

Reactions	$-Q_R(MeV)$	$V_{Bass}(MeV)$	$E_{min}(MeV)$	J_{ER}	<i>Refs.</i>
$^{64}Ni + ^{64}Ni$	48.71	95.7	85.55	-	[3]
$^{60}Ni + ^{89}Y$	90.64	129.9	121.4	-	[5]
$^{90}Zr + ^{92}Zr$	154.0	180.9	169.6	17.0	[4]
$^{28}Si + ^{64}Ni$	1.78	52.3	44.0	-	[8]
$^{16}O + ^{208}Pb$	46.49	76.0	62.7	6.0	[6, 7]

where λ is the multipolarity of the level and the dynamical deformation parameter β_α is taken from independent experiments. In the present calculations the quantity $(\beta_\alpha R_0)/\sqrt{4\pi}$ has been taken constant and the same for all reactions listed in Table I ($\beta_\alpha R_0/\sqrt{4\pi} = 0.3$).

The form of the inner part of the central potential imitating the repulsive interaction is determined, in the present paper, by the expression

$$\Delta V_0(r) = V_1 (R_m - r)^2. \quad (12)$$

which, at $r < R_m$ (R_m is determined by the position of the minimum of the central potential) is similar to one used in Ref. [16]. To compensate the increase of the potential (12) at low distances (increase of $R_m - r$), an additional damping factor

$$g(r) = \left(1 + \exp \frac{r - R_1}{\Delta}\right)^{-1} \quad (13)$$

has been introduced with parameters R_1 Δ as free ones. In such a way the total potential results in a continue function of the distance r with continue first derivatives, as it should be for a nuclear potential. Explicitly, the potential reads as

$$\tilde{V}_0(r) \pm \tilde{V}_{01}(r) = Q_R \cdot g(r) + (V_0(r) \pm V_{01}(r)) \cdot (1 - g(r)). \quad (14)$$

The potential (14) is used to calculate the nuclear penetrability T , eq. (4). Besides penetrability, the fusion cross section is determined by the probability of deexcitation of the

TABLE II: The final values of the phenomenological parameters of the model obtained by fitting the experimental data. The quantities V , R and r_0 are the magnitude of the barrier, its position and nuclear parameter for Coulomb and inner potentials, respectively.

Reactions	V_B	R_B	r_0^B	$\hbar\omega_B$	V_C	R_C	r_0^C	$\hbar\omega_C$
$^{64}\text{Ni} + ^{64}\text{Ni}$	93.55	11.28	1.41	3.57	85.11	6.68	0.84	12.17
$^{60}\text{Ni} + ^{89}\text{Y}$	129.65	11.50	1.37	4.33	122.33	6.92	0.83	9.76
$^{90}\text{Zr} + ^{92}\text{Zr}$	173.63	12.35	1.37	3.46	178.23	7.87	0.87	10.56
$^{28}\text{Si} + ^{64}\text{Ni}$	51.26	10.35	1.47	3.92	45.05	6.04	0.86	19.7
$^{16}\text{O} + ^{208}\text{Pb}$	74.47	11.98	1.42	4.76	61.86	7.44	0.88	14.83

compound system via fission and evaporation processes. In a compound system formed by light nuclei the evaporation process prevails, while for heavy components the fission reaction dominates. In the medium region both processes are of an equal importance. To distinguish between these two processes of deexcitation we use a simplest model, suggested in [25]. It consists in merely multiplication of the penetration probabilities in(4) and (9) by the "deexcitation" factor

$$W_{ER}(L) = \left(1 + \exp \frac{L - J_{ER}}{2}\right)^{-1}, \quad (15)$$

where angular momentum J_{ER} separates fission and evaporation residue cross sections, cf. Table I.

Having determined the form of the cross section we fitted the free parameters to obtain a good description of data in a large region of energies and nuclear masses. The results of our analysis are shown in Table II where the fitted values of the relevant parameters of the model are listed. The main result from the Table can be formulated as follow: since for all combinations of the colliding ions the difference $R_B - R_C$ is almost constant ($\approx 4.5 fm$) the distnace between two barriers is approximatively the same for all the considered reactions. Moreover, as expected $R_B - R_C$ is roughly twice the diffuse edge of nuclei.

As an example, in Figs. 1 we present results of the cross section calculated (solid lines) within our model with parameters listed in Table II for two combinations of colliding nuclei. Since in such kind of reactions the colliding nuclei are considered as light, the theoretical cross section is compared with the experimentally measured evaporation residue cross sections. It

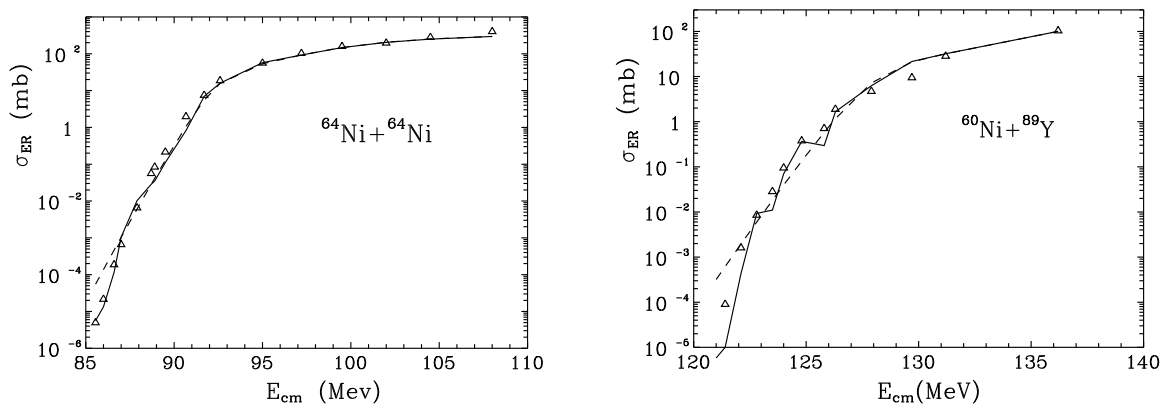


FIG. 1: The calculated cross sections Eq. (3) for the reactions $^{64}\text{Ni} + ^{64}\text{Ni}$, left panel and for the reaction $^{60}\text{Ni} + ^{89}\text{Y}$, right panel. Solid curves correspond to full calculations with including the interference effects in penetrability, eq. (4); dashed curves are the results of calculations without interferences, eq. (10). Experimental data correspond to the measured evaporation residues [3, 5].

can be seen that within two-barrier model a good description of data can be achieved in the whole interval of considered energies for both combinations of colliding ions. It should be pointed out that the hindrance effect has been obtained only by an accurate account of the interference effects, as above discussed. Without interference, i.e. cross section by formula (10) (dashed lines in Figs. 1) an increase of the slope of the cross section at low energies cannot be obtained. Our results are in an agreement with the ones of Ref. [24], the fusion cross sections for the reactions $^{96}\text{Zr} + ^{124}\text{Sn}$, $^{86}\text{Kr} + ^{123}\text{Sb}$, $^{50}\text{Ti} + ^{208}\text{Pb}$, where $\eta > \eta_{cr}$ have been analyzed.

As an illustration of the obtained potential in Fig. 2 we plot the fusion potential without the Coulomb barrier, $V(r) - V_B$ as a function of the distance from the Coulomb barrier $r - R_B$ for five combinations of the colliding particles. It is clearly seen that the position of the barriers relative to R_B is roughly the same. This is an important result since if so, predictions for other sets of colliding nuclei can be easily performed.

Another important result obtained during the phenomenological fit is that at $E = Q_R$ the phase ν in eq. (7) becomes $\approx \pi/2$, which is a clear evidence of the fact that in the inverse to the fusion channel the decaying system was in its first quasi-stationary state. It means that the obtained potential (14) can be successfully applied in describing the decay of heavy nuclei into heavy clusters.

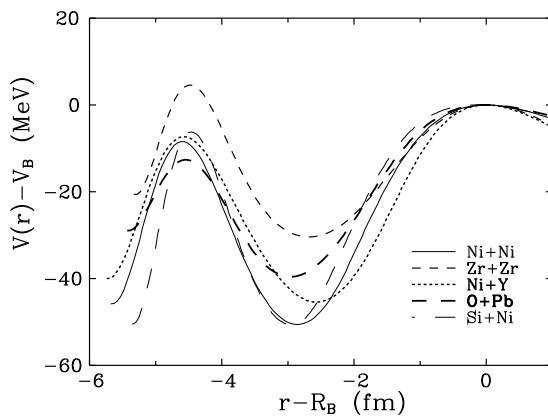


FIG. 2: The phenomenological potential, eq. (12), as a function of the "scaling" variable $r - R_B$ for a set of colliding ions. The scaling-like behaviour of the inner potential is clearly seen at $r - R_B \simeq -4.5 fm$.

IV. CONCLUSION

The fusion processes of two nuclei have been analyzed within a model with complex shaped ion-ion potential. It has been shown that the observed hindrance effect in the fusion cross section at extremely low energies can be described by a two-barrier potential. The corresponding shape and values of the phenomenological parameters have been obtained by a fit of experimental data and it has been shown that the potential depends rather on the relative distance from the Coulomb barrier than the distance between the colliding ions. It has been found that the interference contribution in to the penetrability T plays a crucial role in understanding the drastic change of the slope of the cross section at extremely sub-barrier energies. The obtained phenomenological potential possesses a scaling-like behaviour, in sense that it is almost the same for different ions at the same values of the distance from the Coulomb barrier. This allows to apply our model to other kind of reactions, e.g. the heavy cluster decay processes.

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